ρ = density of continuous phase
σ = constant defined by Equation (25)
$\underline{\underline{\tau}}$ = stress tensor
ψ = stream function

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Part II. Swarm of Bubbles in a Power Law Fluid

The Sherwood number and the drag coefficient for a swarm of bubbles moving in a power law fluid are obtained by an approximate solution of equations of motion in the creeping flow regime. The effect of gas holdup on the motion and mass transfer of a swarm of bubbles with a mobile interface moving in a power law fluid was obtained using Happel's free surface cell model. Since the presence of surfactants may cause small bubbles to behave as solid spheres, expressions for Sherwood number and drag coefficient are also obtained by using the results of power-law fluid motion over assemblages of solid spheres. It is predicted that, unlike the case of a single bubble, the drag coefficient correction factor and the mass transfer correction factor decrease with increasing pseudoplasticity.

SCOPE

Mass transfer in gas dispersions in non-Newtonian fluids is important in wide range of chemical processes, for example, fermentation, sewage treatment, direct contact blood oxygenation, removal of monomers from polymers during the finishing stages, etc. An adequate understanding of the problem of single bubble motion in non-Newtonian fluids together with pertinent experimental data has been available (see Part I). However, information on multiple bubble motion and mass transfer in non-Newtonian fluids is not available. The hydrodynamic field around a bubble is affected significantly by the presence of other bubbles. The purpose of this work is to study the effect of gas holdup and pseudoplasticity on the motion and mass transfer of multiple bubbles under creeping flow conditions.

CONCLUSIONS AND SIGNIFICANCE

The rise velocity of a swarm of bubbles with a mobile interface, moving in a power law fluid, is found to decrease with increasing gas holdup, as found in the case of Newtonian continuous phase. The ratio of swarm velocity to single-bubble velocity for $n = 1$ reduces to the results obtained by Gal-Or and Waslo (1968). Unlike the results for single-bubble motion, increasing pseudoplasticity is found to increase the swarm velocity. The same results were obtained for gas bubbles with immobilized interfaces moving in power law fluids. The effect of pseudoplasticity found in the present work is in agreement with the results obtained by Mohan and Raghuraman (1976) for the motion of an assemblage of solid spheres.

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The correction factor for mass transfer from a swarm of bubbles moving in a power law fluid decreases with increasing pseudoplasticity for both mobile and immobile interface cases.

The effect of pseudoplasticity found in the rise velocity, as well as mass transfer, for both mobile and immobile interfaces, is contrary to the effects found for the case of single-bubble motion, where increasing pseudoplasticity increases the mass transfer and decreases the rise velocity. The physics behind this effect, which is also obtained by Mohan and Raghuraman (1976) for the motion of solid spheres, is not clear.

The effect of gas holdup on motion of and mass transfer from a gas dispersion in Newtonian liquids under creeping flow conditions has been extensively studied by Gal-Or and co-workers. Happel (1958) was the first to develop the free surface cell model for creeping flow of a Newtonian fluid over an assemblage of solid spheres. Later, Gal-Or and Waslo (1968) extended Happel's cell model for the study of creeping motion of a swarm of drops or bubbles in a Newtonian fluid. Ishii and Johnson (1970) used the cell model to study the same problem under potential flow conditions. Marrucci (1965) obtained drag and rise velocity of a swarm of bubbles in potential flow regime using the rate of energy dissipation inside the cell.

Hirose and Moo-Young (1969) have obtained an approximate solution of the equations of motion for single-bubble motion in power law fluids. To obtain the analytical solution, they assumed that the value of the second invariant of the deformation tensor was not too different from its value evaluated using Newtonian velocity profiles. The purpose of the present work is to obtain an approximate solution of the equations of motion for the motion of a swarm of bubbles moving in a power law fluid by following the approach of Hirose and Moo-Young (1969) and by using Happel's free surface cell model.

BUBBLES WITH A MOBILE INTERFACE

Consider the steady, incompressible, creeping flow of a power law fluid past a swarm of gas bubbles, with a superficial velocity of U, in the positive Z direction as shown in Figure 1. Let the bubble radius be R. Then, following Happel's cell model, the cell radius will be equal to $R\phi^{-1/3}$. The equation of motion for this case takes the form

$$\frac{\partial \tilde{p}}{\partial \tilde{r}} = \frac{1}{\tilde{r}^2}\frac{\partial}{\partial \tilde{r}}(\tilde{r}^2\tilde{\tau}_{rr}) + \frac{1}{\tilde{r}\sin\theta}\frac{\partial}{\partial\theta}(\tilde{\tau}_{r\theta}\sin\theta) + \frac{\tilde{\tau}_{\theta\theta}+\tilde{\tau}_{\phi\phi}}{\tilde{r}} \quad (1)$$

$$\frac{1}{\tilde{r}}\frac{\partial \tilde{p}}{\partial\theta} = \frac{1}{\tilde{r}^2}\frac{\partial}{\partial \tilde{r}}(\tilde{r}^2\tilde{\tau}_{r\theta}) + \frac{1}{\tilde{r}\sin\theta}\frac{\partial}{\partial\theta}(\tilde{\tau}_{\theta\theta}\sin\theta) + \frac{\tilde{\tau}_{r\theta}-\tilde{\tau}_{\phi\phi}\cot\theta}{\tilde{r}} \quad (2)$$

where

$$\tilde{\tau}_{ij} = -m\left[\frac{\tilde{\Pi}_d}{2}\right]^{(n-1)/2}\tilde{d}_{ij} \quad (3)$$

and

$$\tilde{\Pi}_d = 4\left\{\left(\frac{\partial \tilde{v}_r}{\partial \tilde{r}}\right)^2 + \left(\frac{1}{\tilde{r}}\frac{\partial \tilde{v}_\theta}{\partial\theta} + \frac{\tilde{v}_r}{\tilde{r}}\right)^2 + \left(\frac{\tilde{v}_r}{\tilde{r}} + \frac{\tilde{v}_\theta\cot\theta}{\tilde{r}}\right)^2\right\} + 2\left\{\tilde{r}\frac{\partial}{\partial \tilde{r}}\left(\frac{\tilde{v}_\theta}{\tilde{r}}\right) + \frac{1}{\tilde{r}}\frac{\partial \tilde{v}_r}{\partial\theta}\right\}^2 \quad (4)$$

Substituting Equation (3) for $\tilde{\tau}_{ij}$ in Equations (1) and (2) and rewriting in terms of the stream function, we get

$$\frac{\partial \tilde{p}}{\partial \tilde{r}} = m\left[\frac{\tilde{\Pi}_d}{2}\right]^{\frac{(n-1)}{2}}\left[-\frac{1}{\tilde{r}^2\sin\theta}\frac{\partial}{\partial\theta}\tilde{E}^2\tilde{\psi}\right.$$

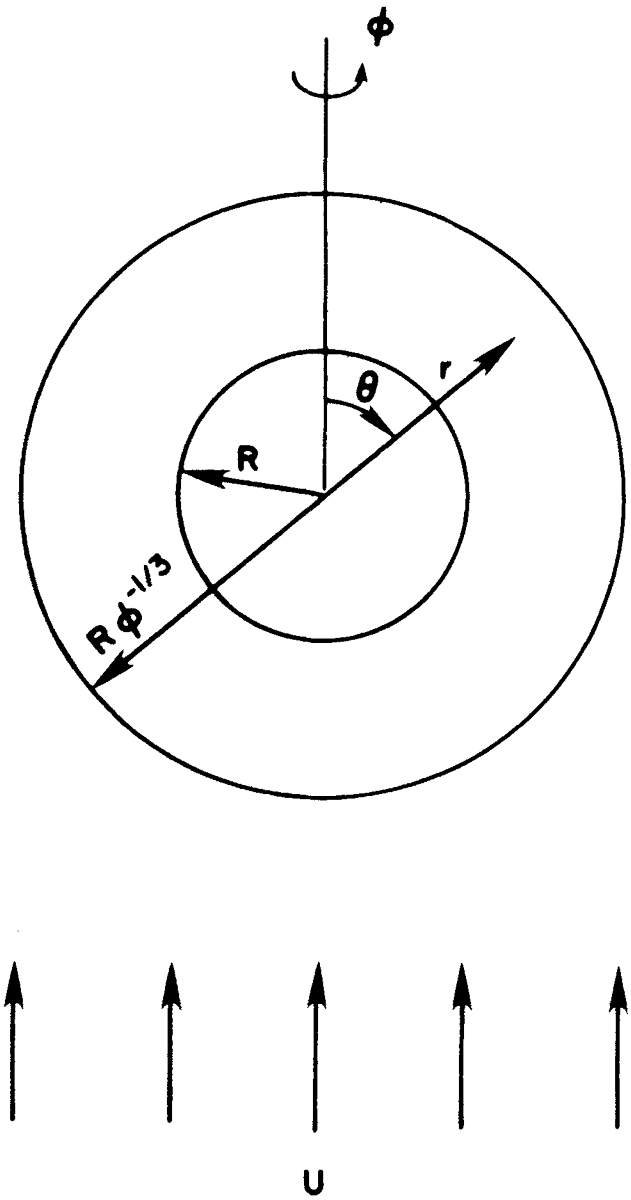

Fig. 1. Coordinate system for the cell model.

Table 1

n		r	$\left[\frac{\tilde{\Pi}_d}{\tilde{\Pi}_{d0}{}^0}\right]^{\frac{n-1}{2}}$
0.9	30°	1	1.004
		2	1.020
		3	1.030
	60°	1	1.004
		2	1.019
		3	1.030
0.8	30°	1	1.011
		2	1.069
		3	1.100
	60°	1	1.011
		2	1.066
		3	1.100
0.7	30°	1	1.018
		2	1.118
		3	1.160
	60°	1	1.018
		2	1.106
		3	1.140

$$+\frac{n-1}{2}\left\{2\frac{\partial\tilde{v}_r}{\partial\tilde{r}}\frac{\partial\ln\tilde{\Pi}}{\partial\tilde{r}}+\frac{1}{\tilde{r}}\left(\tilde{r}\frac{\partial}{\partial\tilde{r}}\frac{\tilde{v}_\theta}{\tilde{r}}+\frac{1}{\tilde{r}}\frac{\partial\tilde{v}_r}{\partial\theta}\right)\frac{\partial\ln\tilde{\Pi}}{\partial\theta}\right\}\Bigg] \quad (5)$$

$$\frac{1}{\tilde{r}}\frac{\partial\tilde{p}}{\partial\theta}=m\left[\frac{\tilde{\Pi}_d}{2}\right]^{\frac{(n-1)}{2}}\left[\frac{1}{\tilde{r}\sin\theta}\frac{\partial}{\partial\tilde{r}}\tilde{E}^2\tilde{\psi}+\frac{n-1}{2}\left\{\left(\tilde{r}\frac{\partial}{\partial\tilde{r}}\frac{\tilde{v}\theta}{\tilde{r}}+\frac{1}{\tilde{r}}\frac{\partial\tilde{v}_r}{\partial\theta}\right)\frac{\partial\ln\tilde{\Pi}}{\partial\tilde{r}}+\frac{1}{\tilde{r}^2}\left(\frac{\partial\tilde{v}}{\partial\theta}+\tilde{v}_r\right)\frac{\theta\ln\tilde{\Pi}}{\partial\theta}\right\}\right] \quad (6)$$

where

$$\tilde{E}^2=\frac{\partial^2}{\partial\tilde{r}^2}+\frac{\sin\theta}{\tilde{r}^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta} \quad (7)$$

and

$$\tilde{v}_r=-\frac{1}{\tilde{r}^2\sin\theta}\frac{\partial\tilde{\psi}}{\partial\theta} \quad (8)$$

$$\tilde{v}_\theta=\frac{1}{\tilde{r}\sin\theta}\frac{\partial\tilde{\psi}}{\partial\tilde{r}} \quad (9)$$

Some approximations will be made following Hirose and Moo-Young (1969). For not too large deviations from Newtonian behavior, that is, $|n-1|<<1$, $[2\tilde{\Pi}_d]^{(n-1)/2}$ is not very different from $[2\tilde{\Pi}_d{}^0]^{(n-1)/2}$, where $\tilde{\Pi}_d{}^0$ is the value of $\tilde{\Pi}_d$ evaluated using Newtonian profiles. Now, since the second terms in Equations (5) and (6) are small, $[2\tilde{\Pi}_d]^{(n-1)/2}$ can be replaced by $[2\tilde{\Pi}_d{}^0]^{(n-1)/2}$ as a first approximation. To get the limits on the deviation of n from unity for which such an approximation is valid, $[\tilde{\Pi}_d/\tilde{\Pi}_d{}^0]^{(n-1)/2}$ is tabulated for various values of n in Table 1, where $\tilde{\Pi}$ is calculated using the results of Hirose and Moo-Young (1969). It can be seen from the table that for $|n-1|\leqq 0.3$, the error is negligible.

Hence, substituting $\tilde{\Pi}_d{}^0$ for $\tilde{\Pi}_d$ in Equations (5) and (6) and eliminating pressure and rewriting in terms of the nondimensional variables

$$r=\tilde{r}/R \quad \psi=\tilde{\psi}/UR^2$$

we obtain

$$E^4\psi-\frac{n-1}{r}\left[2\frac{\partial}{\partial r}+\frac{\tan\theta}{r}\frac{\partial}{\partial\theta}\right]E^2\psi=\frac{6n(n-1)}{r^3}\sin^2\theta \quad (10)$$

This equation may be solved by usual methods (Happel and Brenner, 1965), giving

$$\psi=\sin^2\theta\left[A_1r^{2n+2}+A_2r^2-A_3r+A_4r^{-1}+\frac{6n(n-1)}{(2n+1)}\left\{\left(\frac{1}{(2n+1)}+\frac{1}{8}\right)r+\frac{r}{2}\ln r\right\}\right] \quad (11)$$

The constants A_1, A_2, A_3, and A_4 are evaluated using the boundary conditions proposed by Happel (1958) for the free surface cell model:

$$\begin{array}{lll} 1.\ \tilde{v}_r=0 & \text{at} & \tilde{r}=R \\ 2.\ \tilde{\tau}_{r\theta}=0 & \text{at} & \tilde{r}=R \\ 3.\ \tilde{\tau}_{r\theta}=0 & \text{at} & \tilde{r}=R\Phi^{-1/3} \\ 4.\ \tilde{v}_r-U\cos\theta=0 & \text{at} & \tilde{r}=R\Phi^{-1/3} \end{array} \quad (12)$$

Thus, solving for A_1, A_2, A_3, and A_4 in Equation (11) using the boundary conditions enlisted in (12) we obtain

$$A_1=\frac{6n(n-1)}{4n(2n+1)^2(1-\phi^{1/3})}\left[\frac{1-\Phi^{-2/3}}{1-\Phi-(2n+3)/3}\right] \quad (13)$$

$$A_2=\frac{6n(n-1)\Phi^{1/3}}{(2n+1)(1-\Phi^{1/3})^2}\left[\frac{(\Phi^{-2/3}-1)}{12}-\frac{\ln\Phi^{-1/3}}{2}+\frac{1}{4n(2n+1)}\left(\frac{1-\Phi^{-2/3}}{1-\Phi-(2n+3)/3}\right)\left\{(1-\Phi^{-(2n+1)/3})+\frac{n(2n+1)}{3}\Phi^{-(2n+1)/3}\cdot(1-\Phi^{-2/3})\right\}\right]-\frac{1}{2(1-\Phi^{1/3})} \quad (14)$$

$$A_3=\frac{6n(n-1)}{(2n+1)(1-\Phi^{1/3})^2}\left[\frac{1-\Phi^{1/3}}{(2n+1)}+\frac{3-5\Phi^{1/3}}{24}+\frac{\Phi^{-2/3}}{12}-\frac{\Phi^{1/3}\ln\Phi^{-1/3}}{2}+\frac{1}{4n(2n+1)}\left(\frac{1-\Phi^{-2/3}}{1-\Phi^{-(2n+3)/3}}\right)\left\{(1-\Phi^{-2n/3})+\frac{n(2n+1)}{3}\Phi^{-2n/3}(1-\Phi^{-1/3})\right\}\right]-\frac{1}{2(1-\Phi^{1/3})} \quad (15)$$

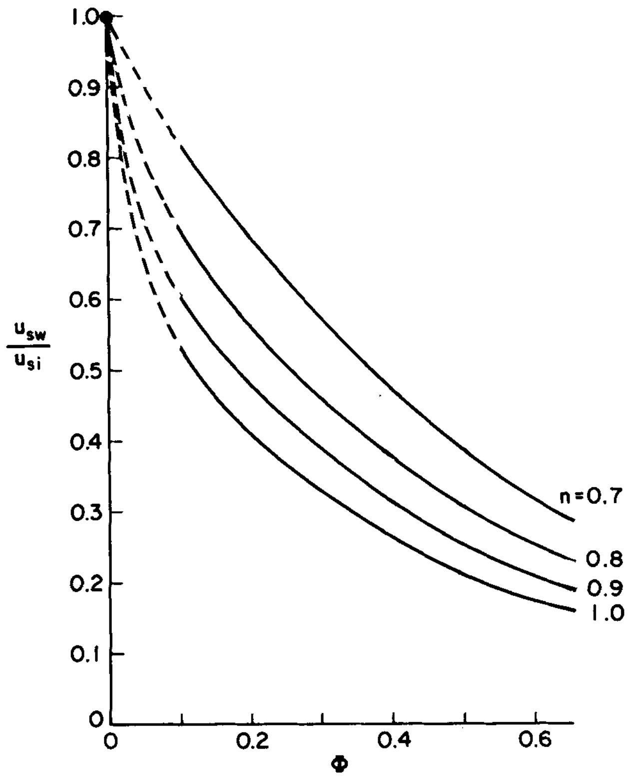

Fig. 2. Ratio of swarm velocity to single-bubble velocity for bubbles with mobile interface.

$$A_4 = \frac{6n(n-1)}{12(2n+1)} \frac{\Phi^{-2/3}}{1-\Phi^{1/3}} \left[\frac{1-\Phi^{-(2n+1)/3}}{1-\Phi^{-(2n+3)/3}} \right] \quad (16)$$

It can be seen that Equation (11) for the stream function reduces to the result of Hirose and Moo-Young (1969) for $\Phi = 0$ and to the Newtonian multiple bubble case for $n = 1$.

DRAG

Substituting for $\tilde{\psi}$ in Equation (6) and integrating, we can evaluate the isotropic pressure ar $r = R$. Similarly, using $\tilde{\psi}$, $\tilde{\tau}_{rr}$ at $\tilde{r} = R$ is also evaluated. Now, if we substitute for $(\tilde{p} + \tilde{\tau}_{rr})$ at $r = R$, in the following definition, the drag force for the motion of multiple bubbles in a power-law fluid is obtained:

$$\text{Drag} = D_{Sw} = -2\pi R^2 \int_0^{\pi} (\tilde{p} + \tilde{\tau}_{rr})_{\tilde{r}=R} \cos\theta \sin\theta \, d\theta$$

$$= 4\pi m 3^{\frac{(n-1)}{2}} R^2 \left(\frac{U}{R}\right)^n Y_{D,Sw} \quad (17)$$

where

$$Y_{D,Sw} = \frac{1}{(1-\Phi^{1/3})^{n-1}} \left[4n(2n+1)A_1 - \frac{2(2n+1)}{n} A_3 + 12A_4 + \left(\frac{n-1}{1-\Phi^{1/3}}\right) \left\{ \frac{1}{n} + \frac{9(1-2n)}{2(1+2n)} \right\} \right] \frac{1}{(n+2)} \quad (18)$$

Since only clean interface is considered here, the frictional drag does not exist, and only contribution to the total drag is from the form drag. It should further be noted that only pseudoplasticity ($n < 1$) is considered in the following.

It can be shown that for $\Phi = 0$ this reduces to Y_D of Hirose and Moo-Young (1969), where

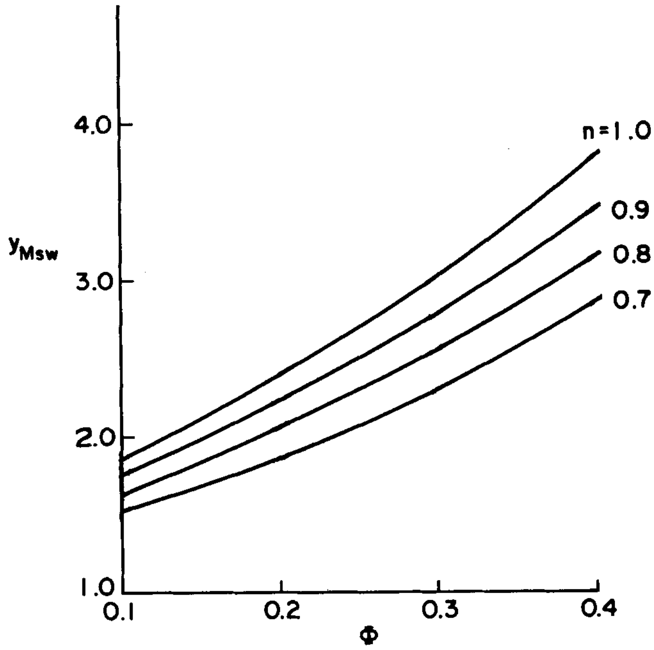

Fig. 3. Correction factor for mass transfer for a swarm of bubbles with mobile interface as a function of gas holdup and power law index.

$$Y_D = \frac{(13 + 4n - 8n^2)}{(2n+1)(n+2)} \quad (19)$$

Thus, the expression for the ratio of swarm velocity to single-bubble velocity in power law fluid can be obtained as

$$\frac{U_{Sw}}{U_{Si}} = \left[\frac{Y_D}{Y_{D,Sw}} \right]^{1/n} \quad (20)$$

This ratio has been plotted in Figure 2 for various values of Φ and n.

MASS TRANSFER

We assume that a thin concentration boundary layer exists near the interface of the bubble, an assumption which is quite justifiable at the high Peclet numbers of interest. Mass transter relation can then be obtained by using the relationship given by Baird and Hamielec (1962):

$$N_{Sh} = \sqrt{\frac{2}{\pi}} \left[\int_0^{\pi} - \frac{\tilde{v}_\theta}{U} \bigg|_{\tilde{r}=R} \sin^2\theta \, d\theta \right]^{1/2} N_{Pe}^{1/2} \quad (21)$$

Substituting for $\tilde{v}_\theta$ obtained from the stream function ψ given by Equation (11) and integrating, we have

$$N_{Sh} = 0.65 \, Y_M N_{Pe}^{1/2} \quad (22)$$

where

$$Y_M = \left[2A_3 + 2A_4 - 2(2n+2)A_1 - 4A_2 - \frac{12n(n-1)}{(2n+1)} \left\{ \frac{1}{2n+1} + \frac{5}{8} \right\} \frac{1}{(1-\Phi^{1/3})} \right]^{1/2} \quad (23)$$

Defining the Peclet number based on the swarm velocity U_{Sw}, we obtain

$$N_{Sh} = 0.65 \, Y_{M,Sw} N_{Pe,Sw}^{1/2} \quad (24)$$

where

$$Y_{M,Sw} = Y_M \Big/ \left[\frac{U_{Sw}}{U} \right]^{1/2} \quad (25)$$

Figure 3 shows $Y_{M,Sw}$ as function of Φ and n.

Table 2

n	Y_D'
1.0	1.00
0.9	1.14
0.8	1.23
0.7	1.27

Table 3

	$Y'_{D,Sw}$			
Φ \ n	1.0	0.9	0.8	0.7
0.6	85.12	56.11	37.03	24.37
0.5	37.91	26.55	18.59	13.00
0.4	18.92	14.03	10.41	7.71
0.3	10.13	7.93	6.21	4.86
0.2	5.64	4.67	3.86	3.20
0.1	3.11	2.75	2.43	2.15

BUBBLES WITH AN IMMOBILIZED INTERFACE

Mohan and Raghuraman (1976) have studied the motion of a power law fluid over an assemblage of solid spheres. Using variational methods, these authors obtained the upper and lower bounds drag and an expression for the stream function.

Defining the correction factors for the drag coefficient for a single sphere and an assemblage of spheres as below

$$C_D = \frac{24}{N_{Re}'} Y_D'$$

and

$$C_{D,Sw} = \frac{24}{N_{Re,Sw}'} Y_{D,Sw}'$$

it can be shown that the ratio of swarm velocity to the single sphere velocity is given by

$$\frac{U_{Sw}'}{U_{Si}'} = \left[\frac{Y_D'}{Y_{D,Sw}'}\right]^{1/n} \qquad (26)$$

Values of Y_D' are obtained from the work of Wasserman and Slattery (1964) and are shown in Table 2. Values of $Y_{D,Sw}'$ are obtained from the work of Mohan and Raghurman (1976) and are shown in Table 3. The velocity ratio thus calculated is shown in Figure 4 as a function of Φ and n.

Mass Transfer

The mass transfer relation for a swarm of bubbles with an immobilized interface can be obtained by using the following expression obtained by Baird and Hamielec (1962):

$$N_{Sh} = 0.641\left[\int_0^\pi \left\{ \left.\frac{\partial \tilde{v}_\theta}{\partial \tilde{r}}\right|_{\tilde{r}=R} \sin\theta \right\}^{1/2} \sin\theta\, d\theta\right]^{2/3} N_{Pe}^{1/3} \qquad (27)$$

Using the following expression obtained by Mohan and Raghuraman (1976), the above integral is evaluated:

$$\psi = [A_1'r^2 + A_2'r^\sigma + A_3'r^{-1} + A_4'r^4]\sin^2\theta \qquad (28)$$

where A_1', A_2', A_3', $A^{4\prime}$, and σ are functions of Φ and n. Thus, we obtain

$$N_{Sh} = 1.0\, Y_M' N_{Pe}^{1/3} \qquad (29)$$

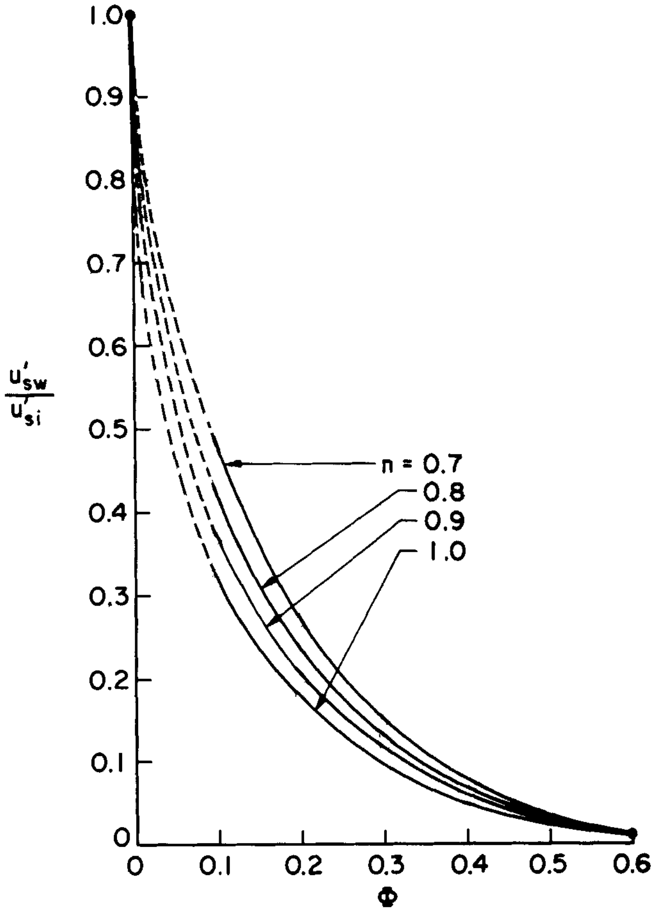

Fig. 4. Ratio of swarm velocity to single-bubble velocity for bubbles with an immobilized interface.

where

$$Y_M' = \left[A_1' + \frac{\sigma(\sigma-1)}{2} A_2' + A_3' + 6\, A^{4\prime}\right]^{1/3} \qquad (30)$$

Defining the Peclet number based on the swarm velocity U_{Sw}'

$$N_{Sh} = 1.0\, Y_{M,Sw}' N_{Pe,Sw}^{1/3} \qquad (31)$$

where

$$Y_{M,Sw}' = Y_M' \Big/ \left[\frac{U_{Sw}'}{U_{Si}'}\right]^{1/3} \qquad (32)$$

Figure 5 shows $Y_{M,Sw}'$ as a function of Φ and n.

DISCUSSION

Motion of a Swarm of Bubbles with Free and Rigid Interfaces

The movement of a single gas bubble rising in the creeping flow regime has been well studied theoretically by Hirose and Moo-Young (1969) and Bhavaraju et al. (1977). Their predictions indicated that the correction factor for drag coefficient (Y_D) for a single bubble is larger for a pseudoplastic fluid than a Newtonian fluid. This observation has been verified with limited data by Hirose and Moo-Young (1969) and with extensive data by Acharya et al. (1977). In view of this, it is somewhat surprising to find that the correction factor for a swarm of bubbles actually decreases with increased pseudoplasticity (decreasing n). The physical implication of this observation is that for a constant phase fraction (Φ); the retarding influence of neighboring gas bubbles on the swarm rise velocity is more in the case of a Newtonian fluid than in the case of a pseudoplastic fluid.

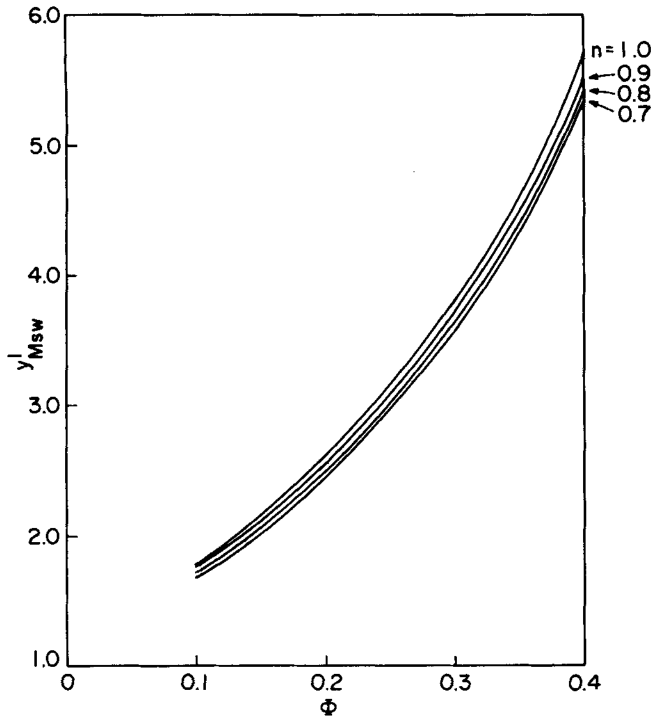

Fig. 5. Correction factor for mass transfer for a swarm of bubbles with an immobilized interface as a function of gas holdup and power law index.

A possible physical explanation for this phenomenon may be as follows. In the case of the motion of a swarm of bubbles, a region in the neighborhood of the bubble (up to the radius of the free surface cell) comprises of a medium with finite and large shear, and this implies lower apparent viscosities and resistance for pseudoplastic fluids in comparison to a Newtonian fluid. There are no data presently available to test the predictions of this model. Ideally, the bubble size as a function of the operating variables (for example, gas rate, rheology, sparger design) can be calculated (Bhavaraju et al. 1977(*b*) and Kumar and Kuloor, 1963). Thus, if bubble size and gas holdup are known, then the swarm rise velocities can be predicted using this model.

Comments concerning the movement of the bubbles with rigid interface are, in principle, similar to the foregoing. Acharya et al. (1976) have provided comprehensive data on the movement of a single sphere in pseudoplastic fluids and verified the predictions that the correction factor (Y_D') for the drag coefficient is greater than unity. The results of Mohan and Raghuraman (1976) show that again for an assemblage of spheres, the trends with increasing pseudoplasticity are in reverse. In this case, an experimental verification of this observation has been provided by Mohan and Raghuraman (1976).

Mass Transfer from a Swarm of Bubbles with Free and Rigid Interfaces

In the case of a single bubble, the increased pseudoplasticity (decreased n) is predicted to increase the value of the Sherwood number in relation to a Newtonian fluid. This has been confirmed experimentally by Hirose and Moo-Young (1969).

The predictions of the present work indicate that for the case of the swarm of bubbles with either free or rigid interfaces, the effect of increased pseudoplasticity is actually reverse in the sense that a pseudoplastic fluid is likely to give a lower Sherwood number than a Newtonian fluid. It should be noted that in the case of a single bubble, the correction factor (Y_D) for the drag coefficient and the correction factor (Y_M) for the mass transfer both increase with increasing pseudoplasticity. The observation of reversal of trends in both the cases of motion and mass transfer for a swarm are internally consistent.

It should be noted that the results obtained for a swarm of bubbles do not reduce to those for a single bubble for $\phi = 0$. The reversal of the trends due to the effect of pseudoplasticity at certain small finite values of ϕ are due to some interaction of pseudoplasticity and holdup, which is unknown at present. Thus, results for $\phi < 0.1$ have not been presented.

The predictions have a pragmatic significance, as increased pseudoplasticity is likely to lead to reduced mass transfer rates in fermentations, where invariably the consistency increases and the pseudoplasticity index falls with the age of the fermentation broth (Tuffle and Pinho, 1970). Experimental data on mass transfer in fermentation broths (for example, Tuffle and Pinho, 1970) show a fall in mass transfer rate with increased age of the broth. However, the data cannot be compared quantitatively with the predictions of this work.

ACKNOWLEDGMENT

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NOTATION

d_{ij} = component of deformation tensor
D_{Sw} = drag force of a swarm of bubbles
$\mathcal{D}$ = molecular diffusivity
k_L = liquid phase mass transfer coefficient
m = consistency coefficient in power law model
n = power law index
N_{Pe} = $U(2R)/\mathcal{D}$, Peclet number
N_{Sh} = $k_L(2R)/\mathcal{D}$, Sherwood number
p = isotropic pressure
r = radial distance from the center of the bubble
R = radius of the bubble
U = approaching velocity far away from the bubble
v_i = velocity component
Y_D, Y_M = correction factors for drag and mass transfer coefficients for single bubble with mobile interface
$Y_{D,Sw}$, $Y_{M,Sw}$ = correction factors for drag and mass transfer coefficients for a swarm of bubbles with a mobile interface
Y_D', Y_M' = correction factors for drag and mass transfer coefficients single bubble with an immobilized interface
$Y_{D,Sw}'$, $Y_{M,Sw}'$ = correction factors for drag and mass transfer coefficients for a swarm of bubbles with an immobilized interface

Greek Letters

Π_d = second invariant of the deformation tensor
ρ = density of continuous phase
τ_{ij} = component of stress tensor
ψ = stream function
Φ = gas holdup

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Semiempirical Theory of Surface Tensions of Pure Normal Alkanes and Alcohols

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A comparison of experimental and theoretical surface tensions leads to a correlation between homogeneous fluid parameters and the inhomogeneous influence parameter *c*. From this correlation, it is possible to predict, to within excellent agreement with experiment, the surface tensions of normal alkanes and alcohols from bulk data only.

SCOPE

The Peng-Robinson equation of state is combined with the mean field theory of inhomogeneous systems to describe both interfacial density profiles and dimensionless surface tensions for normal alkane and alcohol systems. The influence parameter *c* of inhomogeneous fluids is determined from a comparison of experimental and theoretical surface tensions. The parameter is then correlated with homogeneous fluid parameters through a simple relationship having some theoretical meaning and is used to predict the surface tensions of the compounds studied.

CONCLUSIONS AND SIGNIFICANCE

An accurate method of calculating the surface tension of normal alkanes and alcohols has been developed. The calculation is based on the mean field theory for which an equation of state of a homogeneous fluid and values of the influence parameter are required. For the Peng-Robinson equation of state, surface tensions may be computed from the generalized curve for γ^* with the aid of

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